JME 3710 / ME 3710
Final Quiz: December 20, 2011 **Name:** ______________________

Solutions

1. Radiation is exchanged between two cubes that have the same center (Fig. 1, left). The inner and outer cubes are labeled “1” and “2”, respectively. Cube #1 has an edge length of $L_1 = 10\,cm$.

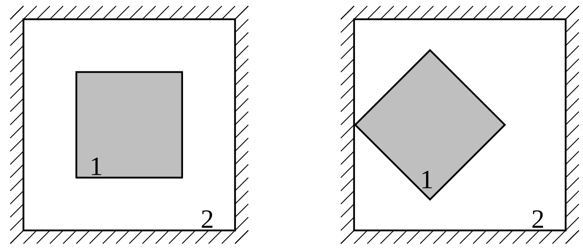


Fig. 1: *Radiation between cubes for 2 configurations*

   (a) (10 pts) For the configuration in the left view, determine the edge length of the outer cube L_2 in units of cm such that the view factor from cube #2 to cube #1 is $F_{21} = 0.64$.

   Solution: By inspection, we see that for the purposes of this configuration, the inner cube is convex, so that $F_{12} = 1$. By reciprocity, we have $A_1 F_{12} = A_2 F_{21}$, for which we can substitute the surface areas to get $6\,L_1^2\,F_{12} = 6\,L_2^2\,F_{21}$. Solving, we find

$$L_2^2 = \frac{6\,L_1^2 F_{12}}{6\,F_{21}} = \frac{10^2 \times 1}{0.64} = 156.25\,cm^2\,,$$

   so that $L_2 = 12.5\,cm$.

   (b) (10 pts) Determine the change in F_{21} (if any) if cube #1 is moved to the position shown in the right–hand panel of Fig. 1. Specifically, cube #1 is centered relative to cube #2 in the “depth dimension” (into the paper), such that its front and back faces do not touch the surfaces of cube #2.

   Solution: Movement does not change the convexity of cube #1, so we still have $F_{12} = 1$. Areas remain the same, so the same reciprocity relationship still holds, implying F_{21} does not change.

2. A long solid rod of diameter $D_1 = 2cm$ dissipates $50W/m$ as a result of carrying an electric current. The rod is situated coaxially inside a tube having an inner diameter $D_2 = 5\,cm$ and whose inner surface is maintained at a uniform temperature $T_2 = 300\,K$ (Fig. 2). The space inside the tube is evacuated such that heat transfer between the rod and the tube can only take place via radiation. All surfaces have emissivities of 0.8.

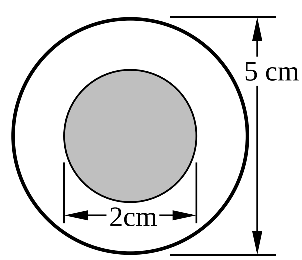
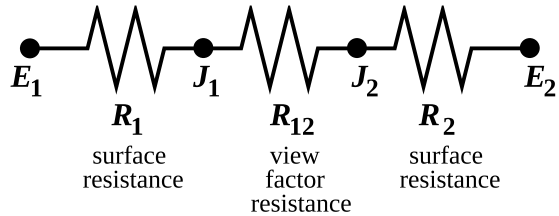


Fig. 2: *Rod and tube enclosure*

   (a) (10 pts) Treating this problem as a two–body enclosure, sketch the radiation circuit analog. Label all emissive powers, radiosities, and briefly describe each of the resistors.

   Solution:



(b) (10 pts) Determine the blackbody emissive power of the inner surface of the tube.

Solution: We compute this directly from the Stefan–Boltzmann Law

$$E_2 \;=\; \sigma\, T_2^4 \;=\; 5.67 \times 10^{-8} \frac{W}{m^2\, K^4}\; 300^4\, K^4 \;\approx\; 459.27\, \frac{W}{m^2}$$

(c) (10 pts) Addressing the problem on a per meter basis, compute the total radiative resistance R_t of the analogous circuit.

Solution: Resistors representing surface effects, i.e. R_1 and R_2 can be calculated from the general formula $R = (1-\varepsilon)/(A\varepsilon)$, which gives

$$R_1 \;=\; \frac{1-0.8}{2\pi \cdot 0.01 \cdot 1 \cdot 0.8}\; \frac{1}{m^2} \;=\; 3.9789\, \frac{1}{m^2}$$

and

$$R_2 \;=\; \frac{1-0.8}{2\pi \cdot 0.025 \cdot 1 \cdot 0.8}\; \frac{1}{m^2} \;=\; 1.5916\, \frac{1}{m^2}\cdot$$

We see by inspection that the view factor from the rod to the tube is $F_{12} = 1$, since the rod is convex and all its radiation must be intercepted by the tube. Therefore, the view factor resistance is

$$R_{12} \;=\; \frac{1}{A_1\, F_{12}} \;=\; \frac{1}{2\pi \cdot 0.01 \cdot 1 \cdot 1}\; \frac{1}{m^2} \;=\; 15.9155\, \frac{1}{m^2}\cdot$$

Since we have a simple series analog, the total resistance is simply the sum of $R_t = R_1 + R_{12} + R_2 \approx 21.486\, m^{-2}$

(d) (10 pts) Calculate the surface temperature of the rod based on circuit analysis.

Solution: The equivalent simplified circuit allows us to write

$$Q \;=\; \frac{E_1 - E_2}{R_t}\,,$$

where $Q = 50$ is the "current". We solve this equation for the blackbody emissive power of the rod

$$E_1 \;=\; Q\, R_t \;+\; E_2 \;=\; 50 \cdot 21.486 \;+\; 459.27 \;=\; 1533.57\, \frac{W}{m^2}\cdot$$

Again, applying the Stefan–Boltzmann equation, $E_1 = \sigma T_1^4$, we solve for the temperature of the rod's surface as

$$T_1 \;=\; \left(\frac{E_1}{\sigma}\right)^{0.25} \;=\; \left(\frac{1533.57}{5.67 \times 10^{-8}}\right)^{0.25} \;\approx\; 405.5\, K\,.$$

3. Water flowing at a rate of $68\, kg/min$ is to be heated from $35\, C$ to $75\, C$ in a simple 1–pass concentric shell–and–tube heat exchanger (counter flow arrangement) by oil, which is to undergo a design temperature drop from $110\, C$ to $75\, C$. Assume specific heats of $4180\, J/(kg\, K)$ for water and $1900\, J/(kg\, K)$ for oil. Also, the overall heat transfer coefficient is known to be $320\, W/(m^2 K)$.

(a) (5 pts) Determine the rate of heat transfer to the water in units of Watts.

Solution: The total heat transfer rate can be determined on the water side as

$$q = \dot{m}_w \, c_w \, \Delta T_w = 68 \cdot 4180 \, (75 - 35) = 11.37 \times 10^6 \; J/min = 1.89 \times 10^5 \; W$$

(b) (5 pts) Find the heat exchanger area in square meters required to effect this rate using the LMTD method of analysis.

Solution: Since all fluid temperatures are known, the LMTD method can be used:

$$\Delta T_{LMTD} = \frac{(110 - 75) - (75 - 35)}{\ln[(110 - 75) \, / \, (75 - 35)]} = 37.44$$

Solve $q = U \, A \, \Delta T_{LMTD}$ for area to obtain

$$A = \frac{1.89 \times 10^5}{320 \cdot 37.44} = 15.8 \, m^2$$

(c) (5 pts) If the diameter of the inner–tube of the heat exchanger is $D = 0.08 \, m$, calculate the necessary length, L, of this device in units of meters. The tube can be assumed to have a "thin" wall.

Solution: The area of heat transfer above is the surface area of the tube, whose formula is $A = \pi D L$, so that

$$L = \frac{A}{\pi \, D} = \frac{15.8}{0.08 \cdot \pi} \approx 62.8 \, m \, .$$

(d) (5 pts) If the design is changed so that the water makes one shell pass and the oil makes two tube passes, calculate the ideal length of the redesigned device (in meters) using the LMTD method. (That is, disregard the turning radius for the oil tube to double–back on itself.) All other parameters remain the same as above. In this *particular* case, a reasonable approximation for the LMTD correction factor (due to the 2–pass arrangement) is

$$F = 1 - 2.665(x - 0.2)^2 \qquad \text{where} \qquad x = \frac{T_{h,o} - T_{h,i}}{T_{c,i} - T_{h,i}} \, .$$

Solution: To solve this part, we employ a correction factor with the LMTD computed on the basis of counterflow conditions, which we already have above. Here,

$$x = \frac{75 - 110}{35 - 110} \approx 0.4667 \quad \text{so that} \quad F = 1 - 2.665(0.4667 - 0.2)^2 \approx 0.81 \, .$$

Therefore,

$$A = \frac{1.89 \times 10^5}{320 \cdot 37.44 \cdot 0.81} \approx 19.46 \, m^2 \, ,$$

and, because there are 2 passes, the overall length is

$$L = \frac{A}{2 \, \pi \, D} = \frac{19.46}{2 \cdot 0.08 \cdot \pi} \approx 38.7 \, m \, .$$

(e) (10 pts) For the configuration in the last sub–part, cross–check the actual heat transfer using the ε–NTU method and compare it to the result obtained from the LMTD method, e.g. by calculating the error percentage.

Solution: The water heat capacity rate is calculated directly from information given in the problem statement

$$(\dot{m}\,c)_w \;=\; \frac{68}{60}\,4180 \;=\; 4737.3\,,$$

while the rate for oil can be deduced from the heat transfer and oil temperature change, as

$$(\dot{m}\,c)_o \;=\; \frac{q}{\Delta T} \;=\; \frac{189000}{110-75} \;=\; 5400\,,$$

whereby the water side is clearly the "min" side and the heat capacity rate ratio is

$$C \;=\; \frac{(\dot{m}\,c)_{min}}{(\dot{m}\,c)_{max}} \;=\; \frac{4737.3}{5400} \;=\; 0.877\,.$$

The maximum theoretical heat transfer is calculated from the definition

$$q_{max} \;=\; (\dot{m}\,c)_{min}\Big(T_{h,i}-T_{c,i}\Big) \;=\; 4737.3(110-35) \;=\; 355300\,,$$

as is the NTU

$$NTU \;=\; \frac{U\,A}{(\dot{m}\,c)_{min}} \;=\; \frac{320\cdot 19.46}{4737.3} \;=\; 1.315\,.$$

The relationship between ε and NTU is calculated from a model *specific* to this particular heat exchanger, a single shell with 2 tube passes. From references:

$$\begin{aligned}
\varepsilon \;&=\; 2\left[1+C+\sqrt{1+C^2}\times\frac{1+\exp(-NTU\sqrt{1+C^2}\;)}{1-\exp(-NTU\sqrt{1+C^2}\;)}\right]^{-1}\\
&=\; 2\left[1+0.877+\sqrt{1+0.877^2}\times\frac{1+\exp(-1.315\sqrt{1+0.877^2}\;)}{1-\exp(-1.315\sqrt{1+0.877^2}\;)}\right]^{-1}\\
&\approx\; 0.531\,.
\end{aligned}$$

The final heat transfer rate is

$$q \;=\; \varepsilon\,q_{max} \;=\; 0.531\cdot 355300 \;\approx\; 188630\,W\,,$$

which is an error of roughly 0.2%.

4. (10 pts) For simple 1–pass concentric–tube heat exchangers configured in the counter–flow arrangement, the applicable ε–NTU relationship is

$$\varepsilon \;=\; \frac{1-e^{-N(1-C)}}{1-C\,e^{-N(1-C)}}\,,$$

where N is the number of transfer units and $C=(\dot{m}c)_{min}/(\dot{m}c)_{max}$ is ratio of the heat capacity rates. Careful inspection will indicate that this formula is valid only for $C<1$, i.e. when there is actually a well–defined "min" side of the heat exchanger. If the rates are identical, as is physically possible, this expression is indeterminate (i.e. one finds a "0/0"–type indeterminacy). Starting with the above expression, prove that the correct formula for the special case of $C = 1$ is

$$\varepsilon \;=\; \frac{N}{N+1} \,.$$

Solution: The general expression is readily shown to be indeterminate for $C = 1$. However, we can use L'Hospitals Rule here

$$\varepsilon \;=\; \lim_{C\to 1}\, \frac{f(C)}{g(C)} \;=\; \lim_{C\to 1}\, \frac{f'(C)}{g'(C)} \,,$$

where we have assigned $f(C)$ and $g(C)$ as the functions representing the numerator and denominator, respectively, of the general formula. (This is similar to the approach of solving the indeterminacy problem for the ΔT_{LMTD} when the differences of temperatures at the inlet and outlet of a heat exchanger are the same.) Taking derivatives, we find

$$f'(C) \;=\; 0 - N\, e^{-N(1-C)} \qquad \text{and} \qquad g'(C) \;=\; 0 - C\, N\, e^{-N(1-C)} - e^{-N(1-C)} \,,$$

so that

$$\lim_{C\to 1}\, \frac{f'(C)}{g'(C)} \;=\; \frac{-\,N}{-\,N-1} \;=\; \frac{N}{N+1} \,,$$

which proves the proposition.